Numerical Analysis II
Qualify Exam

Spring, 2019

Note:

- **Cell phone, laptop or any electronic device with Wi-Fi capability are prohibited.**
- **A simple calculator is allowed.**
- **One 8.5”x11” page note is allowed (2-sided page is fine).**
- **Please show all your work. An answer, correct or incorrect, without proper explanation will yield zero credit.**

1. (**20 points**) Consider solving the IVP: $y' = f(x,y), y(x_0) = y_0$.
   (a) If we view the Trapezoidal method as approximation of $\int_a^b y'(x)dx$ by using the average of two boundary values: $\int_a^b y'(x)dx \approx \frac{b-a}{2}(y'(a) + y'(b))$, construct a multi-step method using the average of three values: two boundary values plus the midpoint value.
   (b) Discuss the consistency, the convergence and the stability of the method developed in (a).

2. **(20 points)** Consider the predictor-corrector method with the Euler’s method as the predictor and the backward Euler’s method as the corrector.
   (a) Is this method A-stable? Please explain your answer.
   (b) Is this method a 2-stage Runge-Kutta method of order 2? Please explain your answer.
   (c) Apply this method to : $y' = x + y,\ \ y(0) = 2$. If $y_0 = 2$, find y_1 and y_2, with step size $h = \frac{1}{2}$.

3. **(30 points)** Consider solving the IVP: $y' = f(x,y), y(x_0) = y_0$. Determine if the following numerical methods are convergent:
   (a) $y_{n+1} = y_n + hf(x_n,y_n) + \frac{h^2}{2}\left(f_x(x_n,y_n) + f_y(x_n,y_n)f(x_n,y_n)\right)$;
   (b) $y_{n+1} = 3y_n - 2y_{n-1} + \frac{h}{2}(f(x_n,y_n) - 3f(x_{n-1},y_{n-1}))$;
   (c) $y_{n+1} = \frac{-3}{2}y_n + 3y_{n-1} - \frac{1}{2}y_{n-2} + 3hf(x_n,y_n)$.

4. **(10 points)** Consider the following 2-stage Runge-Kutta method for solving first order ODE:
   $y' = f(x,y)$
   $y_{n+1} = y_n + h(\gamma_1 k_1 + \gamma_2 k_2)$
   $k_1 = f(x_n,y_n)$
   $k_2 = f(x_n + \alpha h,\ y_n + \beta h k_1)$

   Show that all the Second order 2-stage Runger-Kutta methods share the same absolute stability region.